## SOLUTION OF EXERCISE # 7

# Exercise # 7

**Q.1:** If $A = 200 + j425$ and $B = 150 - j275$, find:

**[a]** $A + B$

**Sol.** $A + B = 200 + j425 + 150 - j275 = \boxed{350 + j150}$

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**[b]** $A - 2B$

**Sol.** $A - 2B = (200 + j425) - 2(150 - j275)$

$= 200 + j425 - 300 + j550 = \boxed{-100 + j975}$

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**Q.2:** Express the following in the form a + jb and $r\angle\theta$:

**[a]** $(5 + j4)^2$

**Sol.** $(5 + j4)^2$

$= (5)^2 + 2(5)(j4) + (j4)^2 = 25 + j40 - 16 = \boxed{9 + j40}$

Here $a = 9$ & $b = 40$

$r = \sqrt{a^2 + b^2}$

$r = \sqrt{(9)^2 + (40)^2}$

$r = \sqrt{81 + 1600}$

$r = \sqrt{1681} = 41$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\theta = \tan^{-1}\left(\frac{40}{9}\right)$

$\boxed{\theta = 77°18'}$

$r\angle\theta = \boxed{41\angle 77°18'}$

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**[b]** $(7 - j2)(4 + j5) + (3 - j)(5 - j2)$

**Sol.** $(7 - j2)(4 + j5) + (3 - j)(5 - j2)$

$= 28 + j35 - j8 - j^2 10 + 15 - j6 - j5 + j^2 2$

$= 28 + 27j + 10 + 15 - 11j - 2 = \boxed{51 + j16}$

Here $a = 51$ & $b = 16$

$r = \sqrt{a^2 + b^2}$

$r = \sqrt{(51)^2 + (16)^2}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

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$r = \sqrt{2601 + 256}$

$r = \sqrt{2857}$

$r = 53.46$

$\theta = \tan^{-1}\left(\frac{16}{51}\right)$

$\theta = 17^\circ 25'$

$$\boxed{r\angle\theta = 53.46\angle 17^\circ 25'}$$

**[c]** $\frac{2+j}{2-j} + \frac{2-j}{2+j}$

**Sol.** $\frac{2+j}{2-j} + \frac{2-j}{2+j} = \frac{(2+j)(2+j)+(2-j)(2-j)}{(2-j)(2+j)}$

$= \frac{4+j2+j2+j^2+4-j2-j2+j^2}{(2)^2-(j)^2}$

$= \frac{8-1-1}{4+1} = \frac{6}{5} = 1.2 = \boxed{1.2+j0}$

Here $a = 1.2$ & $b = 0$

$r = \sqrt{a^2+b^2}$

$r = \sqrt{(1.2)^2+(0)^2}$

$r = 1.2$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\theta = \tan^{-1}\left(\frac{0}{1.2}\right)$

$\theta = 0^\circ$

$$\boxed{r\angle\theta = 1.2\angle 0^\circ}$$

**[d]** $\frac{(3+j2)(5-j3)}{(3-j4)}$ (IA-2016)

**Sol.** $\frac{(3+j2)(5-j3)}{(3-j4)} = \frac{15-j9+j10-j^2 6}{3-j4} = \frac{15+j+6}{3-j4}$

$= \frac{21+j}{3-j4} = \frac{21+j}{3-j4} \times \frac{3+j4}{3+j4} = \frac{63+j84+j3-4}{9+16} = \boxed{\frac{59+j87}{25}}$

Here $a = \frac{59}{25}$, $b = \frac{87}{25}$

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$$r = \sqrt{a^2 + b^2} = \sqrt{\left(\frac{59}{25}\right)^2 + \left(\frac{87}{25}\right)^2}$$

$$r = \sqrt{\frac{3481}{625} + \frac{7569}{625}}$$

$$r = \sqrt{\frac{3481 + 7569}{625}} = \sqrt{\frac{11050}{625}}$$

$$r = 4.20$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta = \tan^{-1}\left(\frac{87}{59}\right)$$

$$\theta = 50°51'$$

$$\boxed{r \angle \theta = 4.20 \angle 55°51'}$$

**Q.3: Express each of the following in Rectangular form i.e., a + jb form:**

**[a] $3 \angle 45°$**

**Sol.** $3 \angle 45°$

$$= 3(\cos 45° + j \sin 45°)$$

$$= 3\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\frac{3\sqrt{2}}{2}(1 + j)}$$

**[b] $86 \angle -115°$**

**Sol.** $86 \angle -115°$

$$= 86\left[\cos(-115°) + j\sin(-115°)\right]$$

$$= 86(-0.4226 - 0.9063j)$$

$$= \boxed{-36.34 - j78}$$

**[c] $2 \angle \frac{\pi}{6}$ rad**

**Sol.** $2\angle\frac{\pi}{6} = 2\angle 30°$

$$= 2\left[\cos 30° + j \sin 30°\right]$$

$$= 2\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = \boxed{\sqrt{3} + j}$$

**[d] $3 \angle 0°$**

**Sol.** $3\angle 0°$

$$= 3[\cos 0° + j \sin 0°]$$

$$= 3(1 + 0j)$$

$$= \boxed{3 + 0j}$$

**Q.4: Evaluate the following expressions:**

**[a] $(5 \angle 45°)(3 \angle 36°)$**

**Sol.** $(5 \angle 45°)(3 \angle 36°) = 5(3) \angle(45° + 36°) = \boxed{15 \angle 81°}$

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**[b]** $(1.1\angle 30^\circ)(2.3\angle 17^\circ)(2.8\angle 74^\circ)$

**Sol.** $(1.1\angle 30^\circ)(2.3\angle 17^\circ)(2.8\angle 74^\circ)$

$=((1.1)(2.3)(2.8))\angle(30^\circ+17^\circ+74^\circ)=\boxed{7.08\angle 121^\circ}$

**[c]** $\dfrac{3.7\angle 17^\circ}{6.5\angle 48^\circ}$

**Sol.** $\dfrac{3.7\angle 17^\circ}{6.5\angle 48^\circ}=\dfrac{3.7}{6.5}\angle(17^\circ-48^\circ)=\boxed{0.5692\angle -31^\circ}$

**[d]** $\dfrac{(8.7\angle 76^\circ)(6.8\angle 62^\circ)(1.2\angle -67^\circ)}{(8.9\angle 74^\circ)(1.9\angle 24^\circ)}$

**Sol.** $\dfrac{(8.7\angle 76^\circ)(6.8\angle 62^\circ)(1.2\angle -67^\circ)}{(8.9\angle 74^\circ)(1.9\angle 24^\circ)}$

$=\dfrac{(8.7)(6.8)(1.2)\angle(76^\circ+62^\circ-67^\circ)}{(8.9)(1.9)\angle(74^\circ+24^\circ)}$

$=\dfrac{70.992\angle 71^\circ}{16.91\angle 98^\circ}=4.198\angle 71^\circ-98^\circ=\boxed{4.2\angle -27^\circ}$

**Q.5: Evaluate the following expressions:**

**[a]** $(1+j)^3$

**Sol.** $(1+j)^3=(1)^3+3(1)^2j+3(1)j^2+(j)^3$

$=1+3j-3-j=\boxed{-2+2j}$

**[b]** $(-2+j3)^4$

**Sol.** $(-2+j3)^4=(-2+j3)^2(-2+j3)^2$

$=((-2)^2+2(-2)(3j)+(3j)^2)\,((-2)^2+2(-2)(3j)+(3j)^2)$

$=(4-12j-9)\,(4-12j-9)$

$=(-5-12j)\,(-5-12j)$

$=25+60j+60j+144j^2$

$=25+120j-144=\boxed{-119+120j}$

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**[c]** $\left[\left(2\angle\frac{\pi}{4}\right)\left(3\angle\frac{\pi}{4}\right)\right]^2$

**Sol.** $\left[\left(2\angle\frac{\pi}{4}\right)\left(3\angle\frac{\pi}{4}\right)\right]^2$

$$=\left[(2\angle45^\circ)(3\angle45^\circ)\right]^2$$

$$=\left[6\angle90^\circ\right]^2$$

$$=(6)^2\angle2(90^\circ)$$

$$=\boxed{36\angle180^\circ}$$

**[d]** $\left(\frac{2\sqrt{2}\angle30^\circ}{\sqrt{3}\angle15^\circ}\right)^2$

**Sol.** $\left(\frac{2\sqrt{2}\angle30^\circ}{\sqrt{3}\angle15^\circ}\right)^2$

$$=\frac{\left(2\sqrt{2}\right)^2\angle2(30)^\circ}{\left(\sqrt{3}\right)^2\angle2(15)^\circ}$$

$$=\frac{8\angle60^\circ}{3\angle30^\circ}$$

$$=\frac{8}{3}\angle(60^\circ-30^\circ)$$

$$=\boxed{2.67\angle30^\circ}$$

**Q.6: Express the following in a + jb from:**

**[a]** $5e^{j0^\circ}$

**Sol.** $5e^{j0^\circ}$

$$=5(\cos0^\circ+j\sin0^\circ)$$

$$=5(1+j(0))=\boxed{5+j0}$$

**[b]** $10e^{j60^\circ}$

**Sol.** $10e^{j60^\circ}$

$$=10(\cos(60^\circ)+j\sin(60^\circ))$$

$$=10\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)=10\left(\frac{1+j\sqrt{3}}{2}\right)$$

$$=5\left(1+j\sqrt{3}\right)=\boxed{5+j5\sqrt{3}}$$

**[c]** $5e^{j\frac{\pi}{3}}$

**Sol.** $5e^{j\frac{\pi}{3}}$

$$=5\left(\cos\frac{\pi}{3}+j\sin\left(\frac{\pi}{3}\right)\right)$$

$$=5\left[\cos60^\circ+j\sin60^\circ\right]$$

$$=5\left[\frac{1}{2}+j\frac{\sqrt{3}}{2}\right]$$

$$=\frac{5}{2}+j\frac{5\sqrt{3}}{2}$$

$$=\boxed{2.5+j4.3}$$

# SOLUTION OF EXERCISE # 7

**Q.7: Find the indicated roots of the following:**

**[a]** $\sqrt{5+j8}$

Sol. Let $Z = 5 + 8j$

Here $a = 5, \quad b = 8$

$$r = \sqrt{a^2 + b^2}$$

$$r = \sqrt{(5)^2 + (8)^2}$$

$$r = \sqrt{25+64}$$

$$r = \sqrt{89}$$

$$\boxed{r = 9.43}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta = \tan^{-1}\left(\frac{8}{5}\right)$$

$$\boxed{\theta = 58^\circ}$$

$$Z = r\angle\theta$$

$$Z = 9.43\angle 58^\circ$$

Taking square root on both sides:

$$Z^{1/2} = (9.43\angle 58^\circ)^{1/2}$$

$$Z^{1/2} = \sqrt{9.43}\angle\frac{1}{2}(58)^\circ$$

In general.

$$Z^{1/2} = 3.07\angle\left(\frac{58^\circ + 360^\circ k}{2}\right)$$

By putting $k = 0, 1$ We will get the require roots.

**[b]** Fifth roots of $-\sqrt{3} - j$

Sol. Let $Z = -\sqrt{3} - j$

Here $a = -\sqrt{3}$, $b = -1$

$$r = \sqrt{a^2 + b^2}$$

$$r = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}$$

$$r = \sqrt{3+1} = \sqrt{4}$$

$$\boxed{r = 2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta = \tan^{-1}\left(\frac{-1}{-\sqrt{3}}\right)$$

$$\boxed{\theta = 210^\circ}$$

$$Z = r\angle\theta$$

$$Z = 2\angle 210^\circ$$

Taking 5th root on both sides:

$$Z^{1/5} = (2\angle 210^\circ)^{1/5}$$

$$Z^{1/5} = (2)^{1/5}\angle\frac{1}{5}(210^\circ)$$

In general.

$$Z^{1/5} = (2)^{1/5}\angle\left(\frac{210^\circ + k360^\circ}{5}\right)$$

By putting $k = 0, 1, 2, 3, 4$, we will get the require roots.

# SOLUTION OF EXERCISE # 7

Q.8: Find all the n of the nth roots of the following:

[a] $Z = 32\angle 45^\circ$ ; $n = 5$

Sol. Let $Z = 32\angle 45^\circ$

$$Z^{1/n} = (32\angle 45^\circ)^{1/n}$$

$$Z^{1/n} = (32)^{1/n}\angle \frac{45^\circ}{n}$$

Put $n = 5$ $\quad Z^{1/5} = (32)^{1/5}\angle \frac{45^\circ}{5}$

In general

$$W_k = Z^{1/5} = (32)^{1/5}\angle\left(\frac{45^\circ + 360^\circ k}{5}\right)$$

$$W_k = Z^{1/5} = 2\angle(9^\circ + 72^\circ k) \rightarrow (i)$$

Taking $k = 0, 1, 2, 3, 4$ eq. (i), we get

$$W_0 = 2\angle(9^\circ + 72^\circ(0)) = 2\angle(9^\circ + 0) = 2\angle 9^\circ$$

$$W_1 = 2\angle(9^\circ + 72^\circ(1)) = 2\angle(9^\circ + 72^\circ) = \boxed{2\angle 81^\circ}$$

$$W_2 = 2\angle(9^\circ + 72^\circ(2)) = 2\angle(9^\circ + 144^\circ) = \boxed{2\angle 153^\circ}$$

$$W_3 = 2\angle(9^\circ + 72^\circ(3)) = 2\angle(9^\circ + 216^\circ) = \boxed{2\angle 225^\circ}$$

$$W_4 = 2\angle(9^\circ + 72^\circ(4)) = 2\angle(9^\circ + 288^\circ) = \boxed{2\angle 297^\circ}$$

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[b] $Z = -16\sqrt{3} + 16j$ ; $n = 5$

Sol. Let $Z = -16\sqrt{3} + 16j$

Here $a = -16\sqrt{3}$ & $b = 16$

$$r = \sqrt{a^2 + b^2}$$

$$r = \sqrt{(-16\sqrt{3})^2 + (16)^2}$$

$$r = \sqrt{1024}$$

$$\boxed{r = 32}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta = \tan^{-1}\left(\frac{16}{-\sqrt{16}\sqrt{3}}\right)$$

$$\boxed{\theta = 150^\circ}$$

## SOLUTION OF EXERCISE # 7

$$Z = r\angle\theta^\circ = 32\angle 150^\circ$$

$$Z^{1/5} = (32\angle 150^\circ)^{1/5}$$

In general

$$W_k = Z^{1/5} = (32)^{1/5} \angle\left(\frac{150^\circ + 360^\circ k}{5}\right)$$

$$W_k = 2\angle(30^\circ + 72^\circ k) \rightarrow (i)$$

Put $k = 0,1,2,3,4$ in eq. (i), we get.

$$W_0 = 2\angle[30^\circ + 72^\circ(0)] = 2\angle[30^\circ + 0] = 2\angle 30^\circ$$

$$W_1 = 2\angle[30^\circ + 72^\circ(1)] = 2\angle[30^\circ + 72^\circ] = \boxed{2\angle 102^\circ}$$

$$W_2 = 2\angle[30^\circ + 72^\circ(2)] = 2\angle[30^\circ + 144^\circ] = \boxed{2\angle 174^\circ}$$

$$W_3 = 2\angle[30^\circ + 72^\circ(3)] = 2\angle[30^\circ + 216^\circ] = \boxed{2\angle 246^\circ}$$

$$W_4 = 2\angle[30^\circ + 72^\circ(4)] = 2\angle[30^\circ + 288^\circ] = \boxed{2\angle 318^\circ}$$

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**Q.9: Given $\omega = u + jv$ and $Z = x + jy$ and $\omega = 3Z^2$, express u and v in terms of x and y.**

**Sol.** As, $\omega = 3Z^2$

$$u + jv = 3(x + jy)^2$$

$$u + jv = 3\left[(x)^2 + (jy)^2 + 2(x)(jy)\right]$$

$$u + jv = 3\left[x^2 - y^2 + 2xyj\right]$$

$$u + jv = 3(x^2 - y^2) + j(6xy)$$

Comparing both sides, we get :

$$\boxed{u = 3(x^2 - y^2), \quad v = 6xy}$$

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**Q.10: The resultant impedance $Z$ of two parallel circuits is given by $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$. If $Z_1 = 5 - j3$ and $Z_2 = 3 + j5$ express $Z$ in the form $a + jb$.**

## SOLUTION OF EXERCISE # 7

**Sol.** $\frac{1}{Z}=\frac{1}{Z_1}+\frac{1}{Z_2}=\frac{1}{5-j3}+\frac{1}{3+j5}$

$$\frac{1}{Z}=\frac{3+j5+5-j3}{(5-j3)(3+j5)}=\frac{8+j2}{15+j25-j9-j^2 15}=\frac{8+j2}{30+j16}$$

$$Z=\frac{30+j16}{8+j2}$$

$$Z=\frac{30+j16}{8+j2}\times\frac{8-j2}{8-j2}=\frac{240-j60+j128-j^2 32}{(8)^2-(j2)^2}$$

$$Z=\frac{272+j68}{64+4}=\frac{\cancel{68}(4+j)}{\cancel{68}}=\boxed{4+j}$$

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**Q.11:** In alternating current theory, the voltage $V$, current $I$ and impedance $Z$ may all be complex number and the basis relation between $V$, $I$ and $Z$ is $I=\frac{V}{Z}$.

Find in $a+jb$ form:

**(i)** $I$ when $V=10$, $Z=4+j3$

**Sol.** $I=\frac{V}{Z}=\frac{10}{4+3j}$

$$I=\frac{10}{4+3j}\times\frac{4-3j}{4-3j}=\frac{40-30j}{(4)^2-(3j)^2}=\frac{40-30j}{16+9}$$

$$I=\frac{40-30j}{25}=\frac{40}{25}-\frac{30}{25}j=\boxed{1.6-1.2j}$$

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**(ii)** $V$ when $I=3+j8$, $Z=10+j5$

**Sol.** $V=IZ$

$$V=(3+j8)(10+j5)$$

$$V=30+j15+j80+j^2 40=30+j95-40=\boxed{-10+j95}$$

## SOLUTION OF EXERCISE # 7

**Q.12: Solve the following equations.**

**[a]** $x^2 = -j16$

**Sol.** $x^2 = -j16$

Here $a = 0$ & $b = -16$

$$r = \sqrt{a^2 + b^2}$$

$$r = \sqrt{(0)^2 + (-16)^2}$$

$$r = \sqrt{0 + 256}$$

$$\boxed{r = 16}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta = \tan^{-1}\left(\frac{-16}{0}\right)$$

$$\boxed{\theta = 270^\circ}$$

$$x^2 = r\angle\theta = 16\angle 270^\circ$$

$$\left(x^2\right)^{1/2} = \left[16\angle 270^\circ\right]^{1/2}$$

$$x = (16)^{\frac{1}{2}}\angle\frac{1}{2}(270^\circ)$$

In general

$$x_k = 4\angle\left[\frac{270^\circ + 360^\circ k}{2}\right]$$

$$x_k = 4\angle\left[135^\circ + 180^\circ k\right] \rightarrow (i)$$

Put k = 0, 1 in eq.(i), we get

$$x_0 = 4\angle\left[135^\circ + 180^\circ(0)\right] = 4\angle\left[135^\circ + 0\right] = \boxed{4\angle 135^\circ}$$

$$x_1 = 4\angle\left[135^\circ + 180^\circ(1)\right] = 4\angle\left[135^\circ + 180^\circ\right] = \boxed{4\angle 135^\circ}$$

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**[b]** $\mathbf{x^5 = 16 - j16\sqrt{3}}$

**Sol.** $x^5 = 16 - j16\sqrt{3}$

Here $a = 16$ & $b = -16\sqrt{3}$

## SOLUTION OF EXERCISE # 7

$$r = \sqrt{a^2 + b^2}$$
$$r = \sqrt{(16)^2 + (-16\sqrt{3})^2}$$
$$r = \sqrt{256 + 768}$$
$$r = \sqrt{1024} \Rightarrow \boxed{r = 32}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$
$$\theta = \tan^{-1}\left(\frac{-16\sqrt{3}}{16}\right)$$
$$\boxed{\theta = 300^\circ}$$

$$x^5 = r\angle\theta = 32\angle 300^\circ$$
$$x = (32\angle 300)^{1/5}$$

In general

$$x_k = 2\angle\left[\frac{300^\circ + 360^\circ k}{5}\right]$$
$$x_k = 2\angle\left[60^\circ + 72^\circ k\right] \rightarrow (i)$$

Putting k = 0, 1, 2, 3, 4 in eq.(i), we get

$$x_0 = 2\angle\left[60^\circ + 72^\circ(0)\right] = 2\angle\left[60^\circ + 0\right] = \boxed{2\angle 60^\circ}$$
$$x_1 = 2\angle\left[60^\circ + 72^\circ(1)\right] = 2\angle\left[60^\circ + 72^\circ\right] = \boxed{2\angle 132^\circ}$$
$$x_2 = 2\angle\left[60^\circ + 72^\circ(2)\right] = 2\angle\left[60^\circ + 144^\circ\right] = \boxed{2\angle 204^\circ}$$
$$x_3 = 2\angle\left[60^\circ + 72^\circ(3)\right] = 2\angle\left[60^\circ + 216^\circ\right] = \boxed{2\angle 276^\circ}$$
$$x_4 = 2\angle\left[60^\circ + 72^\circ(4)\right] = 2\angle\left[60^\circ + 288^\circ\right] = \boxed{2\angle 348^\circ}$$